**AP Calculus Summer Assignment**

*There are two main components of developing an understanding of Calculus. One is an intuitive understanding of the big ideas of Calculus. The other is a level of comfort with mathematical skills that will allow you put those ideas into practice. This summer assignment has two parts that are intended to address those two components.*

**PART 1: Summer Review Packet** (can be found on the following pages of this document)

As you prepare to enter AP Calculus in the fall, there are certain skills you have learned over the years that are essential to developing a solid understanding of calculus. If you do not have these skills, you will find that you will consistently make mistakes in your work even though you understand the Calculus concepts. It can be very frustrating for students when they are tripped up by the algebra and not the calculus. This summer packet is intended to help you brush up and possibly relearn these topics.

You may either print the packet and do the work in the space provided or use a separate sheet and turn in your work in well-labeled sequential order. Show all work for every problem, and do them with the idea of making sure you understand the concepts rather than just getting answers down on the page for credit. If you need to be refreshed in your understanding of any particular topic, I have included some helpful websites that you can use for reference. Also do not rely on the calculator. Half of your AP exam next year is taken without the calculator, so you need to become comfortable without relying on it to do work for you.

These problems are due on the first day of school in August. They will be graded, and you will be tested for mastery of these concepts during the first week of school. You need to get off to a good start, so spend some quality time on this work, but it would be better to wait until the second half of the summer to get it done so that the topics are still fresh on your mind when we begin. There are about 25 topics. Some will go quickly; others will take more time. If you aim to do 2-4 topics a day, the entire packet should not take much more than a week. Waiting until the last minute to begin will not be helpful.

Web resources:

http://www.coolmath.com/precalculus-review-calculus-intro
http://www.purplemath.com/modules/index.htm
http://www.mathematicshelpcentral.com/index.html

**PART 2: Online Lessons Introducing the Fundamental Concepts of Calculus**

This series of lessons is designed help you develop intuition about the big ideas of Calculus. The lessons use analogies and connections to things you already understand to help you draw connections with the concepts we will learn in Calculus. Some of the lessons are very approachable. Others will be more difficult to grasp. All will require a close, thoughtful reading. Don’t plan to do more than a couple of lessons at a time. It is better to let the ideas sink in a little bit before moving on.

Read the lesson once to get an overview of what it is about. Then read it again more closely for details, and write down any questions you or ideas that need to be clarified. Then read it a third time looking for answers to those questions. At the end of each lesson, there are questions. Write your answers to the questions and turn them in with the review packet from Part 1. Most of the lessons have a question that asks for a “grandma-friendly” summary of the main ideas of the lesson. This should require more than a one-sentence answer. Do your best to put the ideas of the lesson into your own words in plain English with specific details and without mathematical jargon.

I hope you will find these lessons interesting. They are a great preview to our entire year of Calculus.

https://betterexplained.com/calculus/lesson-1
AP CALCULUS – Summer Review Packet

Complex Fractions

When simplifying complex fractions, multiply by a fraction equal to 1 which has a numerator and denominator composed of the common denominator of all the denominators in the complex fraction.

Example:

\[
\frac{-7 - \frac{6}{x+1}}{\frac{5}{x+1}} = \frac{-7}{5} \cdot \frac{x+1}{x+1} = \frac{-7x - 7 - 6}{5} = \frac{-7x - 13}{5}
\]

\[
\frac{-2 - \frac{3x}{x-4}}{\frac{5}{x-4}} = \frac{-2}{5} \cdot \frac{x-4}{x-4} = \frac{-2x - 8 + 3x^2}{5(x-4) - 1(x)} = \frac{-2x + 8 + 3x^2}{5x^2 - 20x - x} = \frac{3x^2 - 2x + 8}{5x^2 - 21x}
\]

Simplify each of the following.

1. \(\frac{25 - a}{5 + a}\)

2. \(\frac{2 - \frac{4}{x+2}}{5 + \frac{10}{x+2}}\)

3. \(\frac{4 - \frac{12}{2x-3}}{5 + \frac{15}{2x-3}}\)

4. \(\frac{x - \frac{1}{x} + 1}{x + \frac{1}{x}}\)

5. \(\frac{1 - \frac{2x}{3x-4}}{x + \frac{32}{3x-4}}\)
Function

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: \((f \circ g)(x) = f(g(x))\) OR \(f[g(x)]\) read “f of g of x” Means to plug the inside function (in this case \(g(x)\)) in for x in the outside function (in this case, f(x)).

Example: Given \(f(x) = 2x^2 + 1\) and \(g(x) = x - 4\) find \(f(g(x))\).

\[
f(g(x)) = f(x - 4) \\
= 2(x - 4)^2 + 1 \\
= 2(x^2 - 8x + 16) + 1 \\
= 2x^2 - 16x + 32 + 1 \\
f(g(x)) = 2x^2 - 16x + 33
\]

Let \(f(x) = 2x + 1\) and \(g(x) = 2x^2 - 1\). Find each.

6. \(f(2) = \underline{\phantom{00}}\) \hspace{1cm} 7. \(g(-3) = \underline{\phantom{00}}\) \hspace{1cm} 8. \(f(t + 1) = \underline{\phantom{00}}\)

9. \(f[g(-2)] = \underline{\phantom{00}}\) \hspace{1cm} 10. \(g[f(m + 2)] = \underline{\phantom{00}}\) \hspace{1cm} 11. \(\frac{f(x + h) - f(x)}{h} = \underline{\phantom{00}}\)

Let \(f(x) = \sin x\) Find each exactly.

12. \(f\left(\frac{\pi}{2}\right) = \underline{\phantom{00}}\) \hspace{1cm} 13. \(f\left(\frac{2\pi}{3}\right) = \underline{\phantom{00}}\)

Let \(f(x) = x^2\), \(g(x) = 2x + 5\), and \(h(x) = x^2 - 1\). Find each.

14. \(h[f(-2)] = \underline{\phantom{00}}\) \hspace{1cm} 15. \(f[g(x - 1)] = \underline{\phantom{00}}\) \hspace{1cm} 16. \(g[h(x^3)] = \underline{\phantom{00}}\)
Find \( \frac{f(x + h) - f(x)}{h} \) for the given function \( f \):

17. \( f(x) = 9x + 3 \)  
18. \( f(x) = 5 - 2x \)

Intercepts and Points of Intersection

To find the x-intercepts, let \( y = 0 \) in your equation and solve.  
To find the y-intercepts, let \( x = 0 \) in your equation and solve.

**Example:** \( y = x^2 - 2x - 3 \)

\[
\begin{align*}
\text{x-int. (Let } y &= 0) \\
0 &= x^2 - 2x - 3 \\
0 &= (x - 3)(x + 1) \\
x &= -1 \text{ or } x = 3 \\
x - \text{intercepts } (-1,0) \text{ and } (3,0)
\end{align*}
\]

\[
\begin{align*}
\text{y-int. (Let } x &= 0) \\
y &= 0^2 - 2(0) - 3 \\
y &= -3 \\
y - \text{intercept } (0,-3)
\end{align*}
\]

Find the x and y intercepts for each.

19. \( y = 2x - 5 \)  
20. \( y = x^2 + x - 2 \)

21. \( y = x\sqrt{16 - x^2} \)  
22. \( y^2 = x^3 - 4x \)
Systems

Use substitution or elimination method to solve the system of equations.
Example:
\[ x^2 + y - 16x + 39 = 0 \]
\[ x^2 - y^2 - 9 = 0 \]

**Elimination Method**
\[ 2x^2 - 16x + 30 = 0 \]
\[ x^2 - 8x + 15 = 0 \]
\[ (x - 3)(x - 5) = 0 \]
\[ x = 3 \text{ and } x = 5 \]
Plug \( x = 3 \) and \( x = 5 \) into one original
\[ 3^2 - y^2 - 9 = 0 \]
\[ 5^2 - y^2 - 9 = 0 \]
\[ -y^2 = 0 \]
\[ y = 0 \]
\[ y = \pm 4 \]
Points of Intersection \((5, 4), (5, -4)\) and \((3, 0)\)

**Substitution Method**
Solve one equation for one variable.
\[ y^2 = -x^2 + 16x - 39 \]
(1st equation solved for \( y \))
\[ x^2 - (-x^2 + 16x - 39) - 9 = 0 \]
Plug what \( y^2 \) is equal to into second equation.
\[ 2x^2 - 16x + 30 = 0 \]
(The rest is the same as previous example)
\[ x^2 - 8x + 15 = 0 \]
\[ (x - 3)(x - 5) = 0 \]
\[ x = 3 \text{ or } x = 5 \]

Find the point(s) of intersection of the graphs for the given equations.

23. \[ x + y = 8 \]
\[ 4x - y = 7 \]

24. \[ x^2 + y = 6 \]
\[ x + y = 4 \]

25. \[ x^2 - 4y^2 - 20x - 64y - 172 = 0 \]
\[ 16x^2 + 4y^2 - 320x + 64y + 1600 = 0 \]

Interval Notation

26. Complete the table with the appropriate notation or graph.

<table>
<thead>
<tr>
<th>Solution</th>
<th>Interval Notation</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-2 &lt; x \leq 4)</td>
<td>([-1, 7))</td>
<td>[Graph Image]</td>
</tr>
</tbody>
</table>

[Graph Image]
Solve each equation. State your answer in BOTH interval notation and graphically.

27. \(2x - 1 \geq 0\)  
28. \(-4 \leq 2x - 3 < 4\)  
29. \(\frac{x}{2} - \frac{x}{3} > 5\)

**Domain and Range**

Find the domain and range of each function. Write your answer in INTERVAL notation.

30. \(f(x) = x^2 - 5\)  
31. \(f(x) = -\sqrt{x + 3}\)  
32. \(f(x) = 3\sin x\)  
33. \(f(x) = \frac{2}{x-1}\)

**Inverses**

To find the inverse of a function, simply switch the \(x\) and the \(y\) and solve for the new “\(y\)” value.

**Example:**

\[
f(x) = \sqrt[3]{x+1} \quad \text{Rewrite } f(x) \text{ as } y
\]

\[
y = \sqrt[3]{x+1} \quad \text{Switch } x \text{ and } y
\]

\[
x = \sqrt[3]{y+1} \quad \text{Solve for your new } y
\]

\[
(x)^3 = \left(\sqrt[3]{y+1}\right)^3 \quad \text{Cube both sides}
\]

\[
x^3 = y + 1 \quad \text{Simplify}
\]

\[
y = x^3 - 1 \quad \text{Solve for } y
\]

\[
f^{-1}(x) = x^3 - 1 \quad \text{Rewrite in inverse notation}
\]

Find the inverse for each function.

34. \(f(x) = 2x + 1\)  
35. \(f(x) = \frac{x^2}{3}\)
Also, recall that to PROVE one function is an inverse of another function, you need to show that:
\[ f(g(x)) = g(f(x)) = x \]

**Example:**

If: \( f(x) = \frac{x - 9}{4} \) and \( g(x) = 4x + 9 \) show \( f(x) \) and \( g(x) \) are inverses of each other.

\[
\begin{align*}
g(f(x)) &= 4\left(\frac{x - 9}{4}\right) + 9 \\
&= x - 9 + 9 \\
&= x
\end{align*}
\]

\[
\begin{align*}
f(g(x)) &= \frac{(4x + 9) - 9}{4} \\
&= \frac{4x + 9 - 9}{4} \\
&= \frac{4x}{4} \\
&= x
\end{align*}
\]

\[ f(g(x)) = g(f(x)) = x \] therefore they are inverses of each other.

Prove \( f \) and \( g \) are inverses of each other.

36. \( f(x) = \frac{x^3}{2} \) \hspace{1cm} g(x) = \sqrt[3]{2x} \hspace{1cm} 37. \( f(x) = 9 - x^2, \ x \geq 0 \) \hspace{1cm} g(x) = \sqrt{9 - x} \
Equation of a line

Slope intercept form: \( y = mx + b \)  
Vertical line: \( x = c \) (slope is undefined)

Point-slope form: \( y - y_1 = m(x - x_1) \)  
Horizontal line: \( y = c \) (slope is 0)

38. Use slope-intercept form to find the equation of the line having a slope of 3 and a \( y \)-intercept of 5.

39. Determine the equation of a line passing through the point \((5, -3)\) with an undefined slope.

40. Determine the equation of a line passing through the point \((-4, 2)\) with a slope of 0.

41. Use point-slope form to find the equation of the line passing through the point \((0, 5)\) with a slope of \(\frac{2}{3}\).

42. Find the equation of a line passing through the point \((2, 8)\) and parallel to the line \(y = \frac{5}{6}x - 1\).

43. Find the equation of a line perpendicular to the \(y\)-axis passing through the point \((4, 7)\).

44. Find the equation of a line passing through the points \((-3, 6)\) and \((1, 2)\).

45. Find the equation of a line with an \(x\)-intercept \((2, 0)\) and a \(y\)-intercept \((0, 3)\).
Radian and Degree Measure

Use $\frac{180^\circ}{\pi \text{ radians}}$ to get rid of radians and convert to degrees.

Use $\frac{\pi \text{ radians}}{180^\circ}$ to get rid of degrees and convert to radians.

46. Convert to degrees:  
   a. $\frac{5\pi}{6}$  
   b. $\frac{4\pi}{5}$  
   c. 2.63 radians

47. Convert to radians:  
   a. $45^\circ$  
   b. $-17^\circ$  
   c. $237^\circ$

Angles in Standard Position

48. Sketch the angle in standard position.
   a. $\frac{11\pi}{6}$  
   b. $230^\circ$  
   c. $-\frac{5\pi}{3}$  
   d. 1.8 radians

Reference Triangles

49. Sketch the angle in standard position. Draw the reference triangle and label the sides, if possible.
   a. $\frac{2}{3}\pi$  
   b. $225^\circ$
   c. $-\frac{\pi}{4}$  
   d. $30^\circ$
50. Evaluate each of the following:
   a) \( \sin 180^\circ \)  
   b) \( \cos 270^\circ \)  
   c) \( \sin (-90^\circ) \)  
   d) \( \cos \pi \)  
   e) \( \cos 4\pi \)  
   f) \( \sin (-\pi) \)

**Graphing Trig Functions**

\[ f(x) = \sin(x) \quad f(x) = \cos(x) \]

\( y = \sin x \) and \( y = \cos x \) have a period of \( 2\pi \) and an amplitude of 1. Use the parent graphs above to help you sketch a graph of the functions below. For \( f(x) = A \sin(Bx + C) + K \), \( A = \) amplitude, \( \frac{2\pi}{B} = \) period, \( C \) \( \frac{C}{B} = \) phase shift (positive \( C/B \) shift left, negative \( C/B \) shift right) and \( K = \) vertical shift.

**Graph two complete periods of the function.**

51. \( f(x) = 5 \sin x \)

52. \( f(x) = -\cos 2x \)

53. \( f(x) = -\cos\left(x - \frac{\pi}{4}\right) \)

54. \( f(x) = \cos x - 3 \)
Trigonometric Equations:

Solve each of the equations for $0 \leq x < 2\pi$. Isolate the variable, sketch a reference triangle, find all the solutions within the given domain, $0 \leq x < 2\pi$. Remember to double the domain when solving for a double angle. Use trig identities, if needed, to rewrite the trig functions. (See formula sheet at the beginning of the packet.)

55. $\sin x = -\frac{1}{2}$

56. $2\cos x = \sqrt{3}$

57. $\cos 2x = \frac{1}{\sqrt{2}}$

58. $\sin^2 x = \frac{1}{2}$

59. $\sin 2x = -\frac{\sqrt{3}}{2}$

60. $2\cos^2 x - 1 - \cos x = 0$

61. $4\cos^2 x - 3 = 0$

62. $\sin^2 x + \cos 2x - \cos x = 0$
Inverse Trigonometric Functions:

Recall: Inverse Trig Functions can be written in one of two ways:

\[ \arcsin(x) \quad \text{or} \quad \sin^{-1}(x) \]

Inverse trig functions are defined only in the quadrants as indicated below due to their restricted domains.

\[ \begin{array}{c}
\cos^{-1}x < 0 \\
\sin^{-1}x > 0 \\
\cos^{-1}x > 0 \\
\tan^{-1}x > 0 \\
\sin^{-1}x < 0 \\
\tan^{-1}x < 0
\end{array} \]

Example:
Express the value of “y” in radians.

\[ y = \arctan\left(-\frac{1}{\sqrt{3}}\right) \quad \text{Draw a reference triangle.} \]

This means the reference angle is 30° or \( \frac{\pi}{6} \). So, \( y = -\frac{\pi}{6} \) so that it falls in the interval from \( -\frac{\pi}{2} < y < \frac{\pi}{2} \).

Answer: \( y = -\frac{\pi}{6} \)

For each of the following, express the value for “y” in radians.

106. \( y = \arcsin\left(-\frac{\sqrt{3}}{2}\right) \)
107. \( y = \arccos(-1) \)
108. \( y = \arctan(-1) \)
Example: Find the value without a calculator.

\[ \cos \left( \arctan \frac{5}{6} \right) \]

Draw the reference triangle in the correct quadrant first.

Find the missing side using Pythagorean Thm.

Find the ratio of the cosine of the reference triangle.

\[ \cos \theta = \frac{6}{\sqrt{61}} \]

For each of the following give the value without a calculator.

63. \( \tan \left( \arccos \frac{2}{3} \right) \)

64. \( \sec \left( \sin^{-1} \frac{12}{13} \right) \)

65. \( \sin \left( \arctan \frac{12}{5} \right) \)

66. \( \sin \left( \sin^{-1} \frac{7}{8} \right) \)
Circles and Ellipses

For a circle centered at the origin, the equation is \( x^2 + y^2 = r^2 \), where \( r \) is the radius of the circle.

For an ellipse centered at the origin, the equation is \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \), where \( a \) is the distance from the center to the ellipse along the x-axis and \( b \) is the distance from the center to the ellipse along the y-axis. If the larger number is under the \( y^2 \) term, the ellipse is elongated along the y-axis. For our purposes in Calculus, you will not need to locate the foci.

Graph the circles and ellipses below:

67. \( x^2 + y^2 = 16 \)

68. \( x^2 + y^2 = 5 \)

69. \( \frac{x^2}{1} + \frac{y^2}{9} = 1 \)

70. \( \frac{x^2}{16} + \frac{y^2}{4} = 1 \)
Vertical Asymptotes

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote.

71. \( f(x) = \frac{1}{x^2} \)  
72. \( f(x) = \frac{x^2}{x^2 - 4} \)  
73. \( f(x) = \frac{2 + x}{x^2(1-x)} \)

Horizontal Asymptotes

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is \( y = 0 \).

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Determine all Horizontal Asymptotes.

74. \( f(x) = \frac{x^2 - 2x + 1}{x^3 + x - 7} \)  
75. \( f(x) = \frac{5x^3 - 2x^2 + 8}{4x - 3x^3 + 5} \)  
76. \( f(x) = \frac{4x^5}{x^2 - 7} \)

Laws of Exponents

Write each of the following expressions in the form \( ca^pb^q \) where \( c, p \) and \( q \) are constants (numbers).

75. \( \frac{(2a^2)^3}{b} \)  
76. \( \sqrt[3]{9ab^3} \)  
77. \( \frac{a^{2/p}}{3/a} \)

(Hint: \( \sqrt[3]{x} = x^{1/3} \))

78. \( \frac{ab - a}{b^2 - b} \)  
79. \( \frac{a^{-1}}{(b^{-1})\sqrt{a}} \)  
80. \( \left( \frac{a^2}{b^3} \right)^2 \left( \frac{b^3}{a^2} \right) \)
Laws of Logarithms

Simplify each of the following:

81. \( \log_2 5 + \log_2 (x^2 - 1) - \log_2 (x - 1) \)

82. \( 2 \log_2 9 - \log_2 3 \)

83. \( 3^{2 \log_3 5} \)

84. \( \log_{10} (10^{\frac{1}{2}}) \)

85. \( \log_{10} \left( \frac{1}{10^5} \right) \)

86. \( 2 \log_{10} \sqrt{x} + \log_{10} x^\frac{1}{2} \)

Solving Exponential and Logarithmic Equations

Solve for \( x \). (DO NOT USE A CALCULATOR)

87. \( 5^{x+1} = 25 \)

88. \( \frac{1}{3} = 3^{2x+2} \)

89. \( \log_2 x^2 = 3 \)

90. \( \log_3 x^2 = 2 \log_3 4 - 4 \log_3 5 \)

Factor Completely

91. \( x^6 - 16x^4 \)

92. \( 4x^3 - 8x^2 - 25x + 50 \)

93. \( 8x^3 + 27 \)

94. \( x^4 - 1 \)

Solve the following equations for the indicated variables:

95. \( \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \), for \( a \).

96. \( V = 2(ab + bc + ca) \), for \( a \).

97. \( A = 2\pi r^2 + 2\pi rh \), for positive \( r \).

Hint: use quadratic formula

98. \( A = P + xrP \), for \( P \)

99. \( 2x - 2yd = y + xd \), for \( d \)

100. \( \frac{2x}{4\pi} + \frac{1-x}{2} = 0 \), for \( x \)
Solve the equations for $x$:

101. $4x^3 + 12x + 3 = 0$ 
102. $2x + 1 = \frac{5}{x + 2}$ 
103. $\frac{x + 1}{x} - \frac{x}{x + 1} = 0$

Polynomial Division

104. $(x^5 - 4x^4 + x^3 - 7x + 1) \div (x + 2)$ 
105. $(x^5 - x^4 + x^3 + 2x^2 - x + 4) \div (x^3 + 1)$